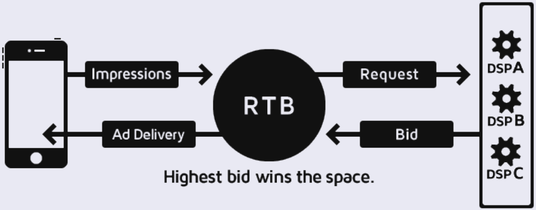
Adaptive On-line Learning for First Bid Prediction and Reserve Price Optimisation

Pedro Chahuara

Real Time Bidding

- Programmatic instantaneous auction to sell/buy Ads
- Two main actors: Demand-side platforms (DSP) supporting buyers and Supply-side plaforms (SSP) supporting publishers

Impressions and advertisement



Final Price

Second bid

- For a given auction: $B = \{b_1 > b_2 > ...b_n\}$, set of sorted bids. r, floor price
- Revenue π of publisher is:

$$\pi(b_1, b_2, r) = \begin{cases} b_2 & \text{if } b_2 \geqslant r \\ r & \text{if } b_1 \geqslant r > b_2 \\ 0 & \text{if } r > b_1 \end{cases} = max(b_2, r)1_{r<b_1}$$

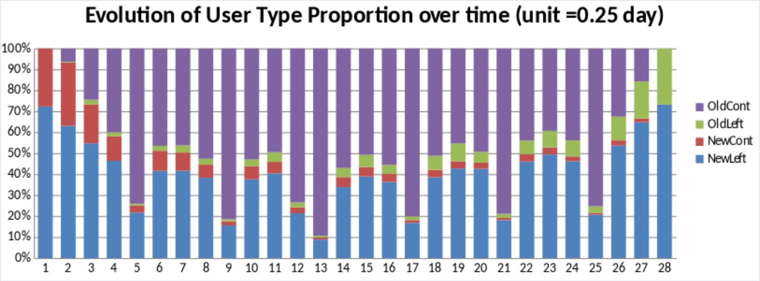
Soft Floor

Some publishers consider also a soft floor, s
$= \pi(b_1, b_2, r, s) = max(b_2, r, min(b_1, s))1_{r<b_1}$

Estimation of the highest bid

Challenges

- Unobserved (censored) data when $B1 < r$
- Data: time, user, tag, highest bid, paid price,reserve price
- No external data(impression, user localisation, user device....)
- "New users", in many cases there is not historical information

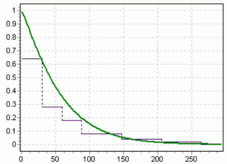


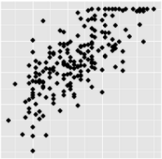
Dealing with censored data

Realiabiliy (survival) function [1]

Probability of an event T not ocurring yet at time T

$S(t) = P(T > t)$





Kaplan Meier Estimator of the survival function $S(t)$

$S(t) = \prod_{t_i < t} \frac{n_i - d_i}{n_i}$

Aalen Addtive Model [Aalen,1980] - linear model with evidence x

$\lambda(t) = b_0(t) + b_1(t)x_1 + b_1(t)x_1, ..., b_n(t)x_n$ with $\lambda(t) = S'(t)/S(t)$

[1] Kaplan, E. L.; Meier, P. (1958). "Nonparametric estimation from incomplete observations".

Tobit Model - Dealing with censored data

- Estimate a latent variable y_i^* given independent x_i
- Observed y_i

$$y_i = \begin{cases} y_i^* & \text{if } y_i^* \geq \tau_i \\ \tau_i & \text{if } y_i^* < \tau_i \end{cases}$$

$$y_i^* = \beta x_i + u_i, u_i \sim N(0, \sigma^2)$$

- log-Likelihood of β
  $log\mathcal{L} = \sum_i log(\frac{1}{\sigma}\phi(\frac{y_i - x_i\beta}{\sigma}))1_{y_i^* \geq \tau_i} + log(\Phi(\frac{x_i\beta - \tau}{\sigma}))1_{y_i^* < \tau_i}$
- Optimal parameters can be found by Newton-Raphson method

Recursive Least Square - Linear Regression

$Y_i^* = \beta X_i + u_i, u_i \sim N(0, \sigma^2)$

- Update parameters β at each observation without requiring the previous set $X_{1..N}$
  $S_N = X'_{1..N} X_{1..N}$
  $S_{N+1} = S_N + X'_{N+1} X_{N+1}$
  $\gamma_{N+1} = S_{N+1}^{-1} + X'_{N+1}$
  $e = Y_{N+1} - X_{N+1} \hat{\beta}_N$
  $\hat{\beta}_{N+1} = \hat{\beta}_N + \gamma_{N+1} e$

- It cannot be directly applied when the dependent variable is censored

Adaptive estimation of a truncated univariate gaussian variable

- predicting $y \sim N(u, \sigma)$
- $\lambda(x) = \frac{\phi(x)}{1-\Phi(x)}$, $\rho(x) = \frac{\phi(x)}{\Phi(x)}$, γ : *factor*

Prediction: $\hat{y} = u$
Update:
If y is not censored :

$$S = \gamma * S + y$$
$$SS = \gamma * SS + (y - u)^2$$

else:

$$M1 = (u - \sigma * \rho((\tau - u)/\sigma))$$
$$S = \gamma * S + M1$$
$$M2 = \sigma^2 + u * M1 - \sigma * \tau * \rho((\tau - u)/\sigma)$$
$$SS = \gamma * SS + (M2 - 2 * u * M1 + u * u)$$

$$u = S/n$$
$$\sigma = \sqrt{SS/n}$$

The Kalman Filter

- A filter to estimate the state of a discrete time dynamic linear system using noisy measurements
- The state of the system at time k is given by:
  $x_{k+1} = A_k x_k + B_k u_k + w_k \quad w_k \sim N(0, Q_k)$
- A measurement:
  $y_k = C_k x_k + v_k \quad v_k \sim N(0, R_k)$
- Conditional pdf of the state:
  $p(x_k \mid Y_k, U_{k-1}) \sim N(\hat{(x)}(k \mid k), P(k \mid k))$
- Kalman Gain:
  $K(k+1) = P(k+1 \mid k)C'_{k+1}[C_{k+1}P(k+1 \mid k)C'_{k+1} + R]^{-1}$

State estimation

- Prediction step
  $\hat{x}(k+1 \mid k) = A_k \hat{x}(k \mid k) + B_k u_k$
  $P(k+1 \mid k) = A_k P(k \mid k) A'_k + Q$
- Filtering step
  $\hat{x}(k \mid k) = \hat{x}(k \mid k-1) + K(k)[y(k) - C_k \hat{x}_{k|k-1}]$
  $P(k \mid k) = [I - K(k)C_k]P(k \mid k-1)$

Kalman Filter for Linear Regression

- Problem definition: $\hat{\beta}_{k+1} = \hat{\beta}_k$
  $y_k = z_k' \hat{\beta}_k + e_k, \;\; e_k \sim N(0, 1)$
- Kalman filter matrices: $A = I, \; R = 0, \; Q = 1, \; C = z_k'$
- Kalman filter equations:
  $K(k+1) = P(k+1 \mid k) z_{k+1}' [z_{k+1} P(k+1 \mid k) z_{k+1}' + 1]^{-1}$
  $\hat{\beta}_{k+1} = \hat{\beta}_k + K(k+1)[y_k - z_k' \hat{\beta}_k]$
  $P(k \mid k) = [I - K(k) z_k] P(k \mid k-1)$

Dynamic Tobit Model [Liao 2014]

- Extension of Kalman Filter to treat truncated data
- Prediction step remains the same:
- Filtering step:
  - $S(k \mid k-1) = C_k P(k \mid k-1) C_k' + R$
  - If Y_k is not censored:
    $\hat{x}(k \mid k) = \hat{x}(k \mid k-1) + K(k)[y(k) - C_k \hat{x}_{k|k-1}]$
    $P(k \mid k) = [I - K(k)C_k]P(k \mid k-1) = P(k \mid k-1) - K(k)S(k \mid k-1)K'(k)$
  - If Y_k is censored:
    $\hat{x}(k \mid k) = \hat{x}(k \mid k-1) + K(k)[E(y(k) \mid y(k) \leq \tau) - C_k \hat{x}_{k|k-1}]$
    $P(k \mid k) = P(k \mid k-1) - K(k)(S(k \mid k-1) - Var(y(k) \mid y(k) \leq \tau))K'(k)$
- In the case of a unvariate variable $\sim N(m, \sigma)$ with
  $m = C_k \hat{x}_{k|k-1}$

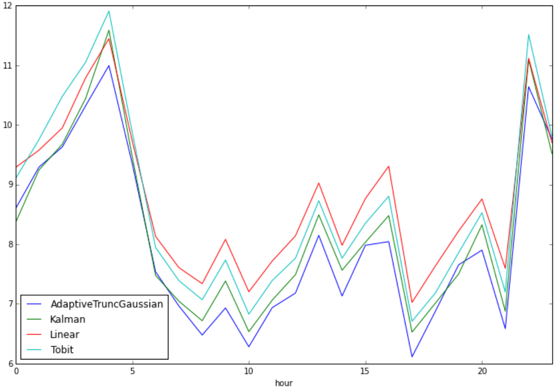
  $$E(y(k) \mid y(k) \leq \tau) = m - \sigma \frac{\phi(\frac{\tau - m}{\sigma})}{\Phi(\frac{\tau - m}{\sigma})}$$

  $$Var(y(k) \mid y(k) \leq \tau) = m^2 - 2m\sigma \frac{\phi(\frac{\tau - m}{\sigma})}{\Phi(\frac{\tau - m}{\sigma})} + \sigma^2(1 - \frac{\tau - m}{\sigma})$$

Some results in Predicting the Highest bid(MAE)

- Features: Previous Highest Price(B1), Previous B1 for same user, Previous B1 for same tag

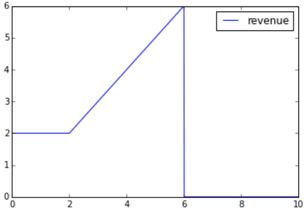
Mean average error per hour:



Revenue Maximization

- The revenue function is asymmetric: underestimating the highest bid is better than overestimating it

- Maximazing the revenue is more than predicting the highest bid



In a second bid setting.

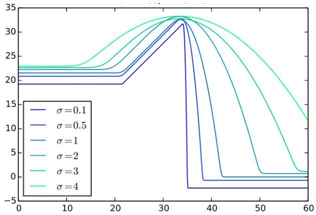
Modeling the reserve price as $f(x_i, w_i) = w' x_i$

Objective: find w that maximize the revenue:

$w^* = argmax \sum_i^N R(f(x_i, w_i), B1_i, B2_i)$

Smoothing the revenue: $y_i \sim N(f(x_i, w_i), \sigma^2)$

$L(w) = \sum_i^N logE[exp(R(y_i, B1_i, B2_i))]$



Revenue Maximization

Application of latent variables [Rudolph 2014]

- Variable z indicates satisfability on the auction

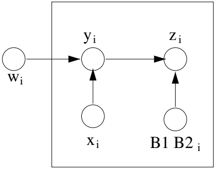
$p(z_i = 1 \mid y_i, B1_i, B2_i) = \pi(y_i, B1_i, B2_i)$

where $\pi(y_i, B1_i, B2_i) = exp(-(B1_i - R(y_i, B1_i, B2_i)))$

The proposed model is:

$y_i \mid w, x_i \sim N(f(x_i, w), \sigma^2)$

$z_i \mid y_i, B1_i, B2_i \sim Bernoulli(\pi(y_i, B1_i, B2_i))$



The smoothed revenue can be mazimized by MAP estimates of w:

$$logp(w \mid z, x, B1, B2) = \sum_i^N (logE[expR(y_i, B1_i, B2_i)] - B1_i)$$

Current work

- Integrating Kalman filter into Revenue Maximization
- Considering non linear models
- Testing on new datasets
- Matrix factorization - Latent variables

- Yuan, Shuai and Wang, Jun and Chen, Bowei and Mason, Peter and Seljan, Sam. An Empirical Study of Reserve Price Optimisation in Real-time Bidding. KDD 14.2014.
- Maja R. Rudolph, Joseph G. Ellis, David M. Blei. Objective Variables for Probabilistic Revenue Maximization in Second-Price Auctions with Reserve. arXiv preprint arXiv:1506.07504.2015